

# Towards a Definition of Topological Correlation: Measuring the Gap Between Multiparameter and Monoparameter Persistent Homology

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## Abstract

Persistent homology is a powerful tool for comparing functions, and it can be approached through two distinct perspectives: the monoparametric view, which analyzes functions component by component, and the multiparametric view, which captures them as a whole. This duality raises a fundamental question: what is the true informative gap between these two approaches, and how can it be understood and measured? At first sight, a seemingly unrelated question also arises when comparing functions: can we determine whether two filtering functions are correlated, and how can such a correlation be quantified? Although these questions may appear independent, we show that they are in fact deeply connected. In this talk, we introduce two new concepts—topological difference and topological correlation—that bring them together within a unified framework. The *topological difference* quantifies the discrepancy between multiparameter and monoparameter persistence, revealing the additional expressive power inherent in the multiparametric setting. Building on this, the *topological correlation* exploits precisely this informative gap to measure the interdependence between filtering functions. Thus, what initially seem like two separate problems turn out to be two sides of the same coin, both rooted in the information that multiparameter persistence retains and the monoparametric one does not. This perspective highlights the strengths of multiparameter persistence and suggests promising directions for applications.

## Keywords

Persistent Homology, Multiparameter Persistence, Topological Data Analysis, Function Correlation, Persistence Modules